J. Fluid Mech. (1991), *vol.* 225, *pp.* 481–495
Printed in Great Britain

Particle dispersion in isotropic turbulence under Stokes drag and Basset force with gravitational settling

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(Received 1 September 1989 and in revised form 24 July 1990)

An analysis that includes the effects of Basset and gravitational forces is presented for the dispersion of particles experiencing Stokes drag in isotropic turbulence. The fluid velocity correlation function evaluated on the particle trajectory is obtained by using the independence approximation and the assumption of Gaussian velocity distributions for both the fluid and the particle, formulated by Pismen & Nir (1978). The dynamic equation for particle motion with the Basset force is Fourier transformed to the frequency domain where it can be solved exactly. It is found that the Basset force has virtually no influence on the structure of the fluid velocity fluctuations seen by the particles or on particle diffusivities. It does, however, affect the motion of the particle by increasing (reducing) the intensities of particle turbulence for particles with larger (smaller) inertia. The crossing of trajectories associated with the gravitational force tends to enhance the effect of the Basset force on the particle turbulence. An ordering of the terms in the particle equation of motion shows that the solution is valid for high particle/fluid density ratios and to $O(1)$ in the Stokes number.

1. Introduction

A particle suspended in homogeneous turbulence responds to the random fluid velocity. As a result, its motion undergoes random displacements that can be characterized statistically by one or more turbulent diffusivity coefficients, by the mean-square velocity fluctuations, and by the mean drift velocity. Calculations presented in the literature, relating the turbulence characteristics of particle motion to the turbulence characteristics of the fluid, have been based on Tchen's (1947) equation or, more recently, on Maxey & Riley's (1983) equation. These relations include the effects of the Stokes drag, the body force, the Basset history force, and the forces due to added mass and local fluid acceleration. Faxen terms that account for local curvature of the velocity field are also included in Maxey & Riley's equation (1983).

Reeks (1977), Pismen & Nir (1978), and Nir & Pismen (1979), used a simplified version of these equations that includes only the Stokes drag and the body force to

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calculate the diffusivities, intensities, and the velocity correlation functions of particle motion in isotropic turbulence. An essential feature of these analyses is that they do not make any *ad hoc* assumptions about $R^{p}_{u_i u_j}(\tau)$, the fluid velocity correlation function evaluated on the particle trajectory. Instead, they incorporate $R^{p}_{u_i u_j}(\tau)$ into a system of equations for calculating the Lagrangian particle velocity correlation $R_{v_i v_j}(\tau)$. Correlations $R^{p}_{u_i u_j}(\tau)$ and $R_{v_i v_j}(\tau)$ are then solved iteratively. The unsteady forces, i.e. the Basset history force and the forces due to added mass and fluid acceleration, are neglected in these analyses because of their complexities. The available results are therefore limited, as the authors have indicated, to cases of a large density ratio $\rho = \rho_p/\rho_f$, where ρ_p and ρ_f are the densities of the particle and the fluid, in addition to the assumption of small particle Reynolds number, $Re = |\boldsymbol{u}^p - \boldsymbol{v}|2a/\nu$, where $\boldsymbol{u}^p$ is the velocity of the fluid seen by the particle, $\boldsymbol{v}$ is the particle velocity, a is the radius of the particle and ν is the kinematic viscosity of the fluid.

Analyses which include the unsteady forces have been presented by Tchen (1947), Chao (1964), Hinze (1975) and, more recently, Gouesbet, Berlemont & Picart (1984). These authors either assume $R^{p}_{u_i u_j}(\tau) = R_{u_i u_j}(\tau)$, where $R_{u_i u_j}(\tau)$ is the Lagrangian fluid velocity correlation function evaluated on the fluid particle trajectory, or use one of Frenkiel's family of correlation functions to describe $R^{p}_{u_i u_j}(\tau)$, thus avoiding the issue of the essential nonlinearity identified by Lumley (1957).

This paper extends the analysis of Pismen & Nir (1978) to include the effect of the unsteady forces, especially the Basset history force. An important ingredient is the determination of the proper ordering of the terms in the dynamic equation of particle motion. Maxey & Riley's equation is non-dimensionalized, after the deterministic settling velocity V_T is subtracted from the total particle velocity V, by using the turbulent root-mean-square velocity, u_0, a typical Eulerian wavenumber, k_0, and a typical Eulerian frequency, ω_0, which characterize the spatial and temporal structure of the turbulence. Only two independent dimensionless parameters result from the non-dimensionalization of the dynamic equation for the fluctuating component of the particle velocity $\boldsymbol{v}$. They are the inertial parameter, $\beta = \frac{9}{2}\nu/(\rho+\frac{1}{2})a^2\omega_0$, which is the ratio of turbulence timescale to the particle response time, and the Stokes number, $\epsilon = (a^2\omega/2\nu)^{\frac{1}{2}}$. The density ratio, ρ, does not appear as an independent parameter. In the resulting equation for $\boldsymbol{v}$, the Stokes drag is of order one, the Basset force is $O(\epsilon)$, and the forces due to the added mass and the local fluid acceleration are $O(\epsilon^2)$. The Stokes number, ϵ, is usually small when the Stokes drag law is used so that the neglect of $O(\epsilon^2)$ in terms involving the fluid velocity fluctuations is justified. A solution is presented for particle dispersion that includes Stokes drag, $O(1)$, the gravitational force, $O(1)$, and the Basset history force, $O(\epsilon)$. The difficulty of solving the dynamic equation in the time domain is avoided by using Fourier transformations, as first suggested by Chao (1964) for this kind of problem. The energy spectrum of the particle velocity, $S_{v_i v_j}(\omega)$, where ω is the dimensionless frequency, is expressed in terms of the spectrum of the fluid velocity seen by the particle, $S^{p}_{u_i u_j}(\omega)$, after the dynamic equation is solved. The fluid velocity correlation seen by the particle, $R^{p}_{u_i u_j}(\tau)$, is the Fourier transformation of $S^{p}_{u_i u_j}(\omega)$. As in Pismen & Nir (1978), this correlation is related to the particle velocity correlation, $R_{v_i v_j}(\tau)$, by means of the independence approximation (or Corrsin's 1959 conjecture) and the Gaussian property of fluctuating velocity of the particle. The closed system of equations is solved iteratively in both the frequency and the time domains by numerical integration. An important feature of this paper is that the influence of the Basset force on the macroscopic behaviour of particle motion is evaluated for a large range of particle inertia and settling rate.

2. Scaling of the governing equation for particle motion

Maxey & Riley (1983) gave the following equation of motion for a sufficiently small spherical particle in unbounded homogeneous turbulence:

$$\begin{aligned}\tfrac{4}{3}\pi a^3\rho_p\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} &= \tfrac{4}{3}\pi a^3(\rho_p-\rho_f)\boldsymbol{g}-6\pi\mu a(\boldsymbol{V}-\boldsymbol{u}^p-\tfrac{1}{6}a^2\nabla^2\boldsymbol{u}^p)\\ &\quad -6\pi\mu a^2\int_{t_0}^{t}\frac{\mathrm{d}}{\mathrm{d}\tau}[\boldsymbol{V}-\boldsymbol{u}^p-\tfrac{1}{6}a^2\nabla^2\boldsymbol{u}^p]\frac{\mathrm{d}\tau}{(\pi\nu(t-\tau))^{\frac{1}{2}}}\\ &\quad -\tfrac{2}{3}\pi a^3\rho_f\frac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{V}-\boldsymbol{u}^p-\tfrac{1}{10}a^2\nabla^2\boldsymbol{u}^p]+\tfrac{4}{3}\pi a^3\rho_f\frac{\mathrm{D}\boldsymbol{u}^p}{\mathrm{D}t}. \end{aligned} \tag{1}$$

In this analysis, the Eulerian fluid velocity at $\boldsymbol{x}$ is denoted by $\boldsymbol{u}(\boldsymbol{x},t)$ and the Lagrangian position and the velocity of the particle are denoted by $\boldsymbol{r}(t)$ and $\boldsymbol{V}(t)$ respectively. The fluid velocity seen by the particle is denoted by $\boldsymbol{u}^p = \boldsymbol{u}(\boldsymbol{r}(t),t)$. It is assumed to have zero mean or, equivalently, the coordinate system is assumed to be moving with a mean fluid velocity that is uniform in space. The terms on the right-hand side of the equation are the gravitational (minus buoyancy) force, the Stokes drag, the Basset history force, the force due to added mass, and the force resulting from the stress gradients of the fluid flow in the absence of a particle. The added mass term is expressed in terms of the time derivative seen by the particle as it moves through its trajectory, $\mathrm{d}/\mathrm{d}t$. The term defining the influence of fluid stress-gradients on the particle is expressed in terms of the change of the fluid velocity along its own trajectory, $\mathrm{D}/\mathrm{D}t$. Equation (1) is valid only when the particle Reynolds number based on the relative velocity between fluid and the particle and particle diameter is very small. The Faxen terms, $a^2\nabla^2\boldsymbol{u}^p$, in (1) are normally small compared with any of the remaining terms, and so they are neglected.

It should be noted here that Auton, Hunt & Prud'homme (1988) have recently found that the added mass term, $(\mathrm{d}/\mathrm{d}t)(\boldsymbol{V}-\boldsymbol{u}^p)$ in (1), should be changed to $(\mathrm{d}\boldsymbol{v}/\mathrm{d}t)-(\mathrm{D}\boldsymbol{u}^p/\mathrm{D}t)$ for inviscid flow over a spherical particle. This change could be especially important in predicting the motion of bubbles in a liquid at high values of Reynolds number, based on the slip velocity. In this study, this possible correction does not carry any tangible significance because the added mass term will be neglected since it is considered a higher-order term than the Basset-force term.

The settling velocity under gravity and buoyancy is given by

$$\boldsymbol{V}_T = \frac{(\rho-1)\boldsymbol{g}}{\frac{9}{2}\nu/a^2} = \lambda u_0\,\boldsymbol{e}_1, \tag{2}$$

where the direction of gravity is given by the unit vector, $\boldsymbol{e}_1$, and $\lambda = V_T/u_0$. If $\boldsymbol{V}_T$ is subtracted from (1), the following equation is obtained for fluctuating velocity $\boldsymbol{v} = \boldsymbol{V}-\boldsymbol{V}_T$:

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = \frac{9}{2}\frac{\nu}{\rho a^2}(u_i^p-v_i)+\frac{9}{2}\frac{\nu}{\rho a^2}\int_{t_0}^{t}\frac{\mathrm{d}}{\mathrm{d}\tau}[u_i^p-v_i]\frac{\mathrm{d}\tau}{(\pi\nu(t-\tau)/a^2)^{\frac{1}{2}}}+\frac{1}{2\rho}\frac{\mathrm{d}}{\mathrm{d}t}[u_i^p-v_i]+\frac{1}{\rho}\frac{\mathrm{D}u_i^p}{\mathrm{D}t}. \tag{3}$$

This equation can then be made dimensionless by introducing

$$\bar{t} = t\omega_0,\quad \bar{\boldsymbol{x}} = \boldsymbol{x}k_0,\quad \bar{\boldsymbol{u}} = \boldsymbol{u}/u_0,\quad \bar{\boldsymbol{v}} = \boldsymbol{v}k_0,\quad \bar{\boldsymbol{k}} = \boldsymbol{k}/k_0,\quad \omega_0 = u_0k_0. \tag{4}$$

The resulting equation is

$$\frac{1}{\bar{\beta}'}\frac{\mathrm{d}\bar{v}_i}{\mathrm{d}\bar{t}} = \bar{u}_i^{\mathrm{p}} - \bar{v}_i + \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \epsilon \int_{t_0}^{t} \frac{\mathrm{d}(\bar{u}_i^{\mathrm{p}} - \bar{v}_i)}{\mathrm{d}\tau} \frac{\mathrm{d}\tau}{(\bar{t}-\tau)^{\frac{1}{2}}} + \tfrac{2}{9}\epsilon^2 \frac{\mathrm{d}(\bar{u}_i^{\mathrm{p}} - \bar{v}_i)}{\mathrm{d}\bar{t}} + \tfrac{4}{9}\epsilon^2 \frac{\mathrm{D}\bar{u}_i^{\mathrm{p}}}{\mathrm{D}\bar{t}}, \tag{5}$$

with
$$\bar{\beta}' = \frac{9}{2}\frac{\nu}{\rho a^2 \omega_0}, \quad \epsilon = \left(\frac{a^2\omega_0}{2\nu}\right)^{\frac{1}{2}}.$$

It is convenient to rearrange (5) so that all terms involving the particle velocity and its derivatives are on the left-hand side, while the fluid velocity related terms are in the right-hand side. Hereinafter, the overbar will be dropped, and all quantities are understood to be dimensionless. This gives

$$\frac{1}{\beta}\frac{\mathrm{d}v_i}{\mathrm{d}t} + v_i + \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \epsilon \int_{t_0}^{t} \frac{\mathrm{d}v_i}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{(t-\tau)^{\frac{1}{2}}} = u_i^{\mathrm{p}} + \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \epsilon \int_{t_0}^{t} \frac{\mathrm{d}u_i^{\mathrm{p}}}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{(t-\tau)^{\frac{1}{2}}} + \tfrac{2}{9}\epsilon^2 \frac{\mathrm{d}u_i^{\mathrm{p}}}{\mathrm{d}t} + \tfrac{4}{9}\epsilon^2 \frac{\mathrm{D}u_i^{\mathrm{p}}}{\mathrm{D}t}, \tag{6}$$

with
$$\beta = \frac{9}{2}\frac{\nu}{(\rho+\frac{1}{2})\, a^2 \omega_0}.$$

The factor β^{-1} is a dimensionless time constant for the response of the particle whose magnitude depends on ϵ as $\beta^{-1} = 4(\rho+\frac{1}{2})\epsilon^2/9$. If the density ratio ρ is of order one or less, the acceleration term on the left-hand side of (6) is of order ϵ^2, the same as the last two acceleration terms on the right-hand side of (9), and all three acceleration terms must be retained. In this case, the term representing acceleration due to fluid stresses $\mathrm{D}u_i^{\mathrm{p}}/\mathrm{D}t$ makes the analysis intractable. If, however, we restrict attention to heavy particles such that the density ratio is of order ϵ^{-1} or larger, then the particle inertial term is of order ϵ, and the two inertia terms on the right-hand side of (6) may be neglected in comparison to it. The factor of $(\rho+\frac{1}{2})$ that appears in β contains a term $(\frac{1}{2})$ that arises from the added mass term. It is neglible in the limit $\rho = O(\epsilon^{-1})$, and ϵ vanishing, but we shall retain it for consistency with notation in earlier work, and for cases in which the numerical value of ϵ is not particularly small so that ρ need not be very large to justify neglecting the added mass acceleration and the fluid stress acceleration.

In the following section (6) will be solved to $O(\epsilon)$ by neglecting all terms of $O(\epsilon^2)$ in the fluctuating fluid velocity. The added mass term on the right-hand side of (6) could be retained, as it is no more difficult mathematically than the Stokes drag and the Basset force terms. However, the last term, $\mathrm{D}\boldsymbol{u}^{\mathrm{p}}/\mathrm{D}t$, which cannot be approximated by $\mathrm{d}\boldsymbol{u}^{\mathrm{p}}/\mathrm{d}t$, is very difficult to deal with analytically. Thus, consistent with the objective of $O(\epsilon)$ in accounting for the influence of the fluid turbulence, all $O(\epsilon^2)$ terms are neglected.

3. Solution of the dynamic equation in the frequency domain

Since solutions are sought for the long-time behaviour, initial conditions for particle velocity are specified for $t \to -\infty$ while the random particle displacement is referenced to its position at $t = 0$ and is evaluated as

$$y_i(t) = \int_0^t v_i(\tau)\,\mathrm{d}\tau. \tag{7}$$

This suppresses any influence of initial conditions on particle dispersion.

Following Chao (1964), the Fourier transformation,

$$\tilde{v}_j(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_j(t)\, \mathrm{e}^{\mathrm{i}\omega t}\, \mathrm{d}t, \tag{8}$$

is applied to (6). After neglecting $O(\epsilon^2)$ terms, one obtains

$$i\frac{\omega}{\beta}\tilde{v}_j(\omega) + \tilde{v}_j(\omega) + \epsilon(1\pm \mathrm{i})|\omega|^{\frac{1}{2}}v_j(\omega) = \tilde{u}^{\mathrm{p}}_j(\omega) + \epsilon(1\pm\mathrm{i})|\omega|^{\frac{1}{2}}\tilde{u}^{\mathrm{p}}_j(\omega), \tag{9}$$

where $(1-\mathrm{i})$ is used for $\omega < 0$. The Fourier component for particle velocity is obtained from the algebraic equation (9) as

$$\tilde{v}_j(\omega) = H(\omega)\,\tilde{u}^{\mathrm{p}}_j(\omega), \tag{10a}$$

where the frequency response function of the particle, $H(\omega)$, is

$$H(\omega) = \frac{1+\epsilon|\omega|^{\frac{1}{2}} \pm \mathrm{i}\epsilon|\omega|^{\frac{1}{2}}}{1+\epsilon|\omega|^{\frac{1}{2}} + \mathrm{i}(\pm\epsilon|\omega|^{\frac{1}{2}} + \omega/\beta)}. \tag{10b}$$

The energy spectrum of particle velocity

$$S_{v_i v_j}(\omega) = \langle \tilde{v}^*_i(\omega)\,\tilde{v}_j(\omega)\rangle, \tag{11}$$

is, therefore,

$$S_{v_i v_j}(\omega) = |H(\omega)|^2 S^{\mathrm{p}}_{u_i u_j}(\omega), \tag{12}$$

where $\langle\ \rangle$ means ensemble average, the symbol * denotes complex conjugate, and $S^{\mathrm{p}}_{u_i u_j}(\omega)$ is the energy spectrum of the fluid velocity seen by the particle. This energy spectrum, $S^{\mathrm{p}}_{u_i u_j}(\omega)$, is also unknown and depends on the particle trajectory due to the essential nonlinearity of the problem. The key step here is to relate $S^{\mathrm{p}}_{u_i u_j}(\omega)$ to the known Eulerian statistics through the particle mean-square displacement. Since the correlation and the spectrum are a Fourier transform pair,

$$\begin{aligned} S^{\mathrm{p}}_{u_i u_j}(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} R^{\mathrm{p}}_{u_i u_j}(\tau)\, \mathrm{e}^{\mathrm{i}\omega\tau}\, \mathrm{d}\tau, \\ S_{v_i v_j}(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{v_i v_j}(\tau)\, \mathrm{e}^{\mathrm{i}\omega\tau}\, \mathrm{d}\tau, \end{aligned} \tag{13}$$

the problem of finding $S^{\mathrm{p}}_{u_i u_j}(\omega)$ is therefore equivalent to finding $R^{\mathrm{p}}_{u_i u_j}(\tau)$.

Following Pismen & Nir (1978), the correlation function, $R^{\mathrm{p}}_{u_i u_j}(\tau)$, is represented for a stationary process as

$$R^{\mathrm{p}}_{u_i u_j}(\tau) = \langle u_i(0,0)\, u_j(\lambda\tau\, \boldsymbol{e}_1 + \boldsymbol{y}(\tau), \tau)\rangle, \tag{14}$$

where $\boldsymbol{y}(\tau) = \boldsymbol{r}(\tau) - \lambda\tau\boldsymbol{e}_1$ is the deviation of the particle position from the deterministic, vertical settling trajectory. By using a Fourier representation of $u_j(\boldsymbol{r}(\tau),\tau)$ in wavenumber space

$$u_j(\boldsymbol{r},\tau) = \int_{-\infty}^{\infty} \hat{u}_j(\boldsymbol{k},\tau)\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}^3\boldsymbol{k}, \tag{15}$$

by using Corrsin's conjecture (1959), and by assuming that the particle velocity is Gaussian, the correlation function, $R^{\mathrm{p}}_{u_i u_j}(\tau)$, can be related to the particle deterministic settling trajectory, $\lambda\tau$, the particle random displacement, Y_{ij}, and the turbulence spectral density function, $\Phi_{ij}(\boldsymbol{k},\tau)$. The result is

$$R^{\mathrm{p}}_{u_i u_j}(\tau) = \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}k_1\lambda\tau}\Phi_{ij}(\boldsymbol{k},\tau)\exp\left(-\tfrac{1}{2}k_i k_j Y_{ij}\right)\mathrm{d}^3\boldsymbol{k}, \tag{16}$$

(see details in Pismen & Nir 1978), where

$$Y_{ij}(\tau) = 2\int_0^{\tau}\int_0^{t'} R_{v_i v_j}(t'')\,\mathrm{d}t''\,\mathrm{d}t' = 2\int_0^{\tau}(\tau-t')\,R^{\mathrm{s}}_{v_i v_j}(t')\,\mathrm{d}t' \tag{17}$$

is the mean square of the random displacement tensor, and $R^{\mathrm{s}}_{v_i v_j}(t')$ is the symmetrical part of the correlation tensor $R_{v_i v_j}(t')$. Equations (12) and (16), together with Fourier transformations (13) now form a closed system defining $S^{\mathrm{p}}_{u_i u_j}(\omega)$, $S_{v_i v_j}(\omega)$, $R^{\mathrm{p}}_{u_i u_j}(\tau)$, $R_{v_i v_j}(\tau)$, and $Y_{ij}(\tau)$ provided $\Phi_{ij}(\boldsymbol{k},\tau)$ is given.

4. Solution for an isotropic, homogeneous turbulence model

The flow is assumed to be homogeneous and isotropic. The spectral density function $\Phi_{ij}(\boldsymbol{k},\tau)$ is taken to be the same form as used by Kraichnan (1970), Phythian (1975), Lundgren & Pointin (1976), Reeks (1977), Pismen & Nir (1978) and Maxey (1987):

$$\Phi_{ij}(\boldsymbol{k},\tau)\,\mathrm{d}^3\boldsymbol{k} = \frac{16}{(2\pi)^{\frac{3}{2}}}k^2\exp\left(-2k^2\right)\left(\delta_{ij}-\frac{k_i k_j}{k^2}\right)\exp^{(-\frac{1}{2}\tau^2)}\,\mathrm{d}^3\boldsymbol{k}. \tag{18}$$

It is noted that $\Phi_{ij}(\boldsymbol{k},\tau)$ is assumed to be separable in $\boldsymbol{k}$ and τ and to decay in time as a Gaussian. Since k and τ in the above expression have been made dimensionless by k_0 and ω_0, it follows that the Eulerian longitudinal integral lengthscale and the Eulerian integral timescale, defined by $\Phi_{ij}(\boldsymbol{k},\tau)$ in (18), are $L_{11} = (2\pi)^{\frac{1}{2}}/k_0$ and $T_0 = (\frac{1}{2}\pi)^{\frac{1}{2}}/\omega_0$, respectively.

Mathematically, the existence and uniqueness of the solution to the problem outlined in the previous section may be difficult to prove for a general $\Phi_{ij}(\boldsymbol{k},\tau)$ because of the highly nonlinear nature of the equations (see (16), for example). Physically, however, it is expected that solutions for the velocity correlations, energy spectra, diffusivities and intensities of the particle motion exist, and that there is only one meaningful solution for an assumed, physically realistic $\Phi_{ij}(\boldsymbol{k},\tau)$.

For turbulent self-diffusion in isotropic flow, the Lagrangian correlation function is of the form $R_{u_i u_j}(\tau) = 0$ for $i \neq j$ and $R_{u_\alpha u_\alpha}(\tau) = R_{u_\beta u_\beta}(\tau)$ for $\alpha \neq \beta$, where the repeated Greek index does not imply summation. For particle dispersion in isotropic turbulence, $R^{\mathrm{p}}_{u_i u_j}(\tau) = 0$ for $i \neq j$ is also expected, and this relation can be used as an initial guess to obtain a solution to the system of equations (Reeks 1977). The assumption of $S^{\mathrm{p}}_{u_i u_j}(\omega) = 0$ for $i \neq j$ implies that $S_{v_i v_j}(\omega) = 0$ and that $R_{v_i v_j}(\tau) = 0$ for $i \neq j$. The exponent in (16) is then $-\frac{1}{2}(k_1^2 Y_{11} + k_2^2 Y_{22} + k_3^2 Y_{33})$. The integration in (16) is thus symmetric with respect to $\boldsymbol{k}$, and $R^{\mathrm{p}}_{u_i u_j}(\tau) = 0$ for $i \neq j$ continues to hold in successive iterations. Therefore,

$$R^{\mathrm{p}}_{u_\alpha u_\alpha}(\tau) = \int_{-\infty}^{\infty}\cos(k_1\lambda\tau)\,\Phi_{\alpha\alpha}(\boldsymbol{k},\tau)\exp\left[-\tfrac{1}{2}k_n^2\,Y_{nn}(\tau)\right]\mathrm{d}^3\boldsymbol{k}, \tag{19}$$

where
$$Y_{\alpha\alpha}(\tau) = 2\int_0^{\tau} (\tau - t')\, R^{\mathrm{s}}_{v_\alpha v_\alpha}(t')\,\mathrm{d}t'. \tag{20}$$

Defining
$$\mu_\alpha = 1 + \tfrac{1}{4} Y_{\alpha\alpha}(\tau), \tag{21}$$

(19) integrates to
$$\left.\begin{aligned} R^{\mathrm{p}}_{u_1 u_1}(\tau) &= \frac{1}{\mu_2^2 \mu_1^{\frac{1}{2}}} \exp\left[-\tfrac{1}{2}\tau^2\left(1 + \frac{\lambda^2}{4\mu_1}\right)\right], \\ R^{\mathrm{p}}_{u_2 u_2}(\tau) &= \tfrac{1}{2} R^{\mathrm{p}}_{u_1 u_1}(\tau)\left[1 + \frac{\mu_2}{\mu_1}\left(1 - \frac{\lambda^2 \tau^2}{4\mu_1}\right)\right]. \end{aligned}\right\} \tag{22}$$

Equations (12), (13) and (20)–(22), together with the pertinent inverse Fourier transformations, form a closed system that can be solved by a successive iteration procedure that converges rapidly. The dimensionless diffusivities and intensities of particle motion are then obtained from

$$D_{\alpha\alpha} = \int_0^{\infty} R_{v_\alpha v_\alpha}(\tau)\,\mathrm{d}\tau = \frac{1}{\pi} S_{v_\alpha v_\alpha}(0) = \frac{1}{\pi} S^{\mathrm{p}}_{u_\alpha u_\alpha}(0), \tag{23}$$

$$\langle v_\alpha^2 \rangle = R_{v_\alpha v_\alpha}(0). \tag{24}$$

The numerical integrations that are required to evaluate (20) and the Fourier transformations are carried out using Gaussian quadrature with a degree of precision of 7. It is of interest to note that the present formulation in terms of both the frequency and the time domains does not involve differential equations while that of Pismen & Nir (1978) results in an integro-differential equation for $Y_{\alpha\alpha}(\tau)$. In the present formulation, β appears only in (12), and the system behaves well as β becomes very large. In Pismen & Nir's (1978) formulation, the integrand in the integro-differential equation behaves like an impulse function as β becomes large, and very small timesteps are needed to solve the equation.

5. Results and discussion

5.1. *Dispersion with zero Basset force* ($\epsilon = 0$)

The effects of inertia and gravity force (or settling velocity) on particle dispersion have been examined by Reeks (1977) and Nir & Pismen (1979). Inertial effects can be represented by the parameter β. Reeks (1977) used a Froude number $F_{\mathrm{g}} = k_0 u_0^2/g$ to represent the gravitational effect and examined the behaviour of particle dispersion in a (β, F_{g})-plane. Nir & Pismen (1979) represent the effect of gravity by the settling rate for a linear drag, $\lambda = 1/(F_{\mathrm{g}}\beta)$. It is felt that the physics of dispersion, is better revealed using (β, λ) because these terms arise naturally from the system of equations governing the dispersion, e.g. (12 and 16); the description of the dispersion is a kinematic issue after the dynamic equation is solved. Results will first be presented for the case when the Basset force is neglected, because Nir & Pismen (1979) did not thoroughly present the solutions in the (β, λ)-plane. Next the case $\epsilon > 0$ is compared with the case $\epsilon = 0$ to allow an assessment of the effect of the Basset force.

Figure 1 shows the dependence of dimensionless particle lateral diffusivities, D_{22}, and intensities, $\langle v_2^2 \rangle$, on $(\beta,\ \lambda)$. It can be seen that in the $(\beta,\ \lambda)$-plane, the settling velocity or mean drift velocity, λ, has a strong influence on D_{22} at any fixed β. However, it is noted that D_{22} is only weakly dependent on inertia. Very similar

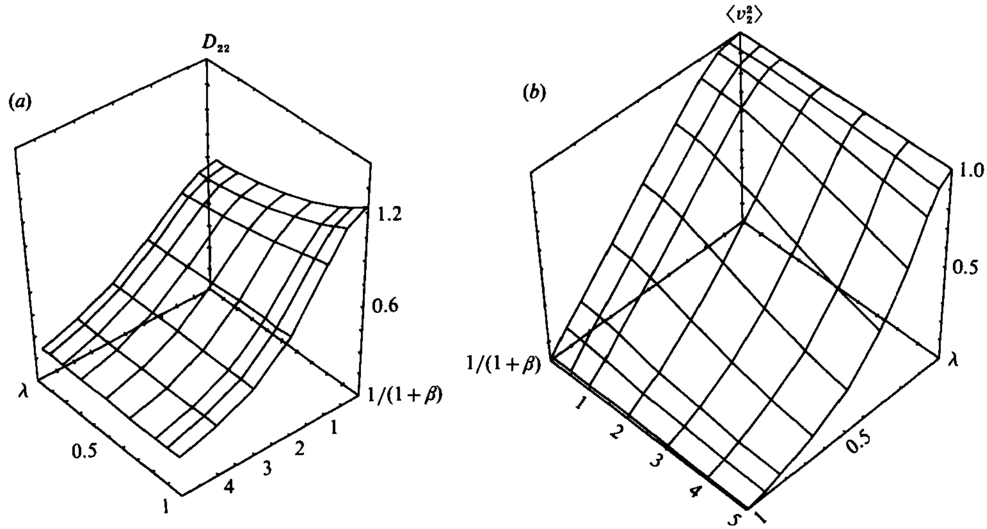


FIGURE 1. Effect of particle inertia and settling rate on particle dispersion for $\epsilon = 0$. Intensities and diffusivities are made dimensionless by turbulence scales u_0 and k_0. (*a*) Lateral diffusivity; (*b*) Lateral intensity.

features are also observed for D_{11}. In fact, D_{11} and D_{22} behave asymptotically as $(2\pi)^{\frac{1}{2}}/\lambda$ and $(2\pi)^{\frac{1}{2}}/2\lambda$ as λ becomes large, showing that D_{11} and D_{22} are asymptotically independent of β at large λ. When $\lambda = 0$, diffusivities D_{11} and D_{22} change at most by 37% from 0.913 (fluid diffusivity) to 1.2533 as β^{-1} varies from 0 to ∞. It is also interesting to consider very small particles with very large values of β (say 50). Such particles respond well to turbulence and have almost the same energy as the turbulence. Their diffusivities, however, will be less than that of the fluid as long as they have significant mean drift (say $\lambda \gtrsim 1$). This is so because a particle with significant mean drift sees a less energetic turbulence at lower frequency than does a particle with no mean drift, which is shown in figure 5, to be discussed later. An equivalent interpretation in the time domain is that the particle with significant settling tends to drift away from strongly correlated turbulent eddies and thus see a less correlated turbulence environment than does a particle with no settling velocity.

The influence of settling velocity on the intensities, $\langle v_\alpha^2 \rangle$, is not so strong as that of inertia, as shown in figure 1(*b*) for $\langle v_2^2 \rangle$. The mean drift, λ, has little effect on $\langle v_\alpha^2 \rangle$ for particles with large β. For particles with small β, $\langle v_\alpha^2 \rangle$ is asymptotically proportional to β as β approaches zero. An effect of settling on $\langle v_\alpha^2 \rangle$ is, however, clearly seen for $\beta \sim O(1)$ and $\lambda > 1$. These results are consistent with those of Reeks (1977).

The results in figure 1(*a*) are also reflected in the correlation functions of fluid velocity seen by the particle, $R^{\mathrm{p}}_{u_\alpha u_\alpha}(\tau)$, presented in figure 2(*a*) for β ranging from 0 to ∞ at $\lambda = 0$ and in figure 2(*b*) for λ between 0 and 5 at $\beta = 1$. As β varies from 0 to ∞, $R^{\mathrm{p}}_{u_\alpha u_\alpha}(\tau)$ changes monotonically from the larger Eulerian fluid velocity correlation to the smaller Lagrangian fluid velocity correlation. This trend is opposite to what was observed in an experiment by Shlien & Corrsin (1974). The fact that the Eulerian correlation is larger than the Lagrangian one in the present analysis and the previous analyses of Kraichnan (1970), Phythian (1975), Lundgren & Pointin (1976), and Reeks (1977) is a natural consequence of the form of $\Phi_{ij}(\boldsymbol{k}, \tau)$

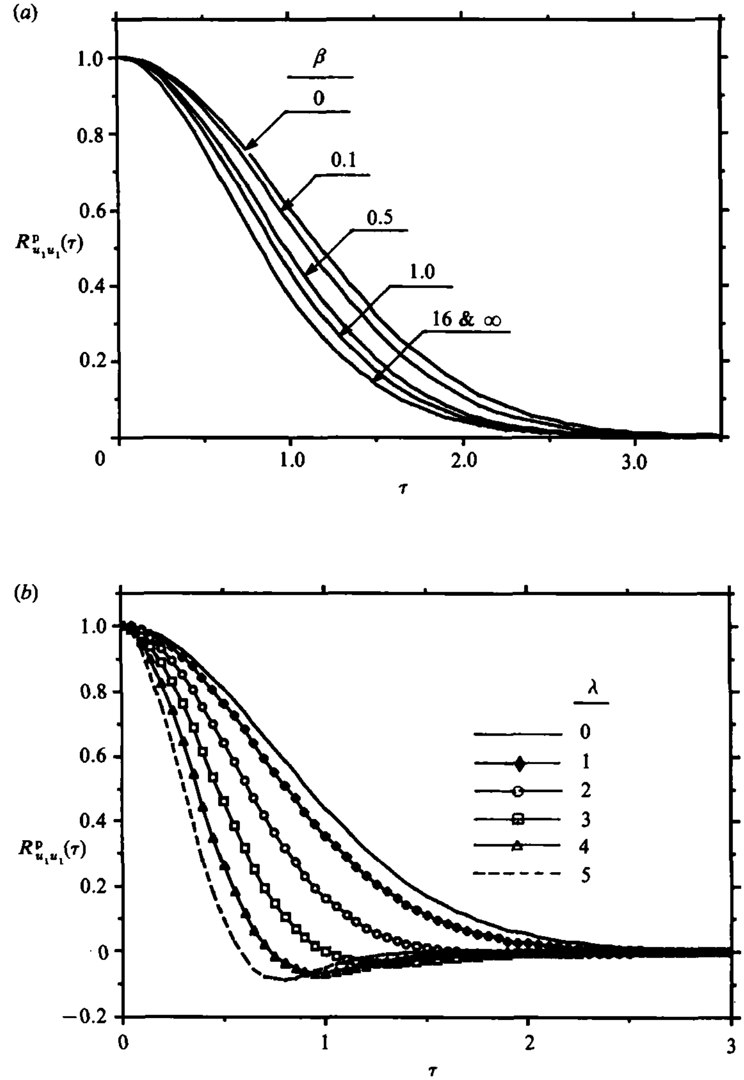


FIGURE 2. (*a*) Effect of particle inertia on the fluid velocity correlation seen by particle $R^{p}_{u_1u_1}(\tau)$ at $\epsilon = 0$. The particle settling velocity is $\lambda = 0$. (*b*) Effect of particle settling velocity on the lateral velocity correlation seen by particle $R^{p}_{u_2u_2}(\tau)$ at $\epsilon = 0$. The particle inertia is $\beta = 1$.

used in the analyses and, probably, the assumption of the Gaussian turbulence. It should have nothing to do with the Corrsin's conjecture because the numerical simulation of fluid particle dispersion by Kraichnan (1970) in a Gaussian turbulence revealed the same trend as using Corrsin's conjecture. The changes in the area under the correlation curves in figure 2(*a*) are not large, implying that the changes in diffusivities due to changes in β are not large. The significant effect of λ on $R^{p}_{u_2u_2}(\tau)$ is clearly demonstrated in figure 2(*b*). As λ increases, negative tails develop in $R^{p}_{u_2u_2}(\tau)$ at $\lambda > 2$. A further increase in λ leads to a more rapid loss of the correlation of $u^{p}_{2}(t)$ and a large decrease in the timescale. Negative tails are inherent in the Eulerian correlation in order to satisfy conservation of mass. The negative values shown in figure 2(*b*) are interpreted as resulting from particles rapidly moving from

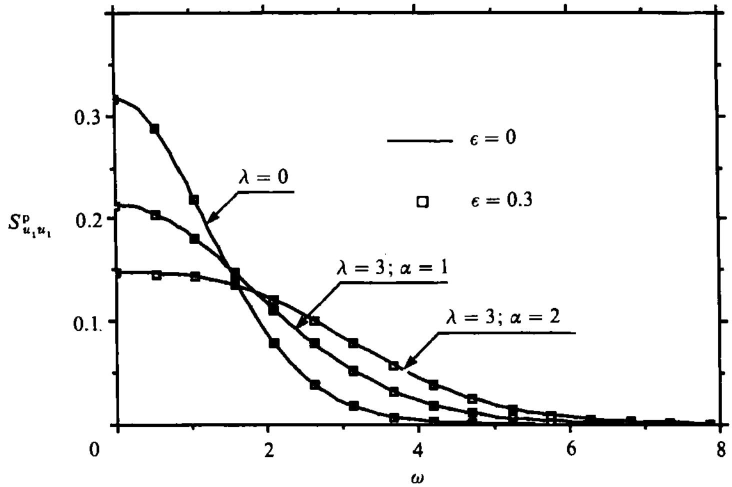


FIGURE 3. Fluid velocity spectrum seen by particle $S^{\rm p}_{u_\alpha u_\alpha}(\omega)$ at $\beta = 1$ for $\epsilon = 0$ and $\epsilon = 0.3$.

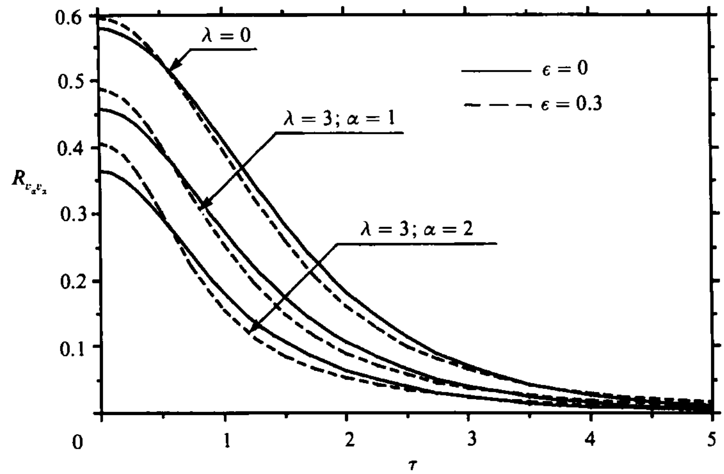


FIGURE 4. Particle velocity correlation $R_{v_\alpha v_\alpha}(\tau)$ at $\beta = 1$, $\lambda = 0$ and for $\epsilon = 0$ and 0.3.

a region of positive (negative) velocity into a region of negative (positive) velocity when λ is large.

5.2. *Effect of the Basset force*

The effect of the Basset force on diffusivities can now be examined by comparing with results for $\epsilon = 0$. It is found that the largest relative difference of lateral diffusivity, $(D_{22}(\epsilon = 0.3) - D_{22}(\epsilon = 0))/D_{22}(\epsilon = 0)$, for $\epsilon = 0.3$ is 0.7 % at $(\beta, \lambda) = (0.2, 0)$ for the whole range of (β, λ) investigated. Even at a larger Stokes number, $\epsilon = 1$, the maximum effect is only 2.7 % at $(\beta,\ \lambda) = (0.5, 0)$. It is clear that the Basset force has little influence on particle diffusivities for all values of $(\beta,\ \lambda)$. Furthermore, it is found that the Basset force changes $R^{\rm p}_{u_\alpha u_\alpha}(\tau)$ by less than 0.003 between $\epsilon = 0$ and $\epsilon = 0.3$ for all values of τ. From figure 3 it is seen that the inclusion of the Basset force at $\epsilon = 0.3$ results in an indistinguishable difference between $S^{\rm p}_{u_\alpha u_\alpha}(\epsilon = 0.3)$ and $S^{\rm p}_{u_\alpha u_\alpha}(\epsilon = 0)$. This implies that the Basset force has virtually no influence on the statistics of the detailed structure of the fluid velocity seen by the particle.

Figures 4 and 5 show particle velocity correlation functions, $R_{v_\alpha v_\alpha}(\tau)$, and energy spectra, $S_{v_\alpha v_\alpha}(\omega)$, at $\beta = 1$ for different values of λ and ϵ. Since $R^{\rm p}_{u_\alpha u_\alpha}(\tau)$ and $S^{\rm p}_{u_\alpha u_\alpha}(\omega)$

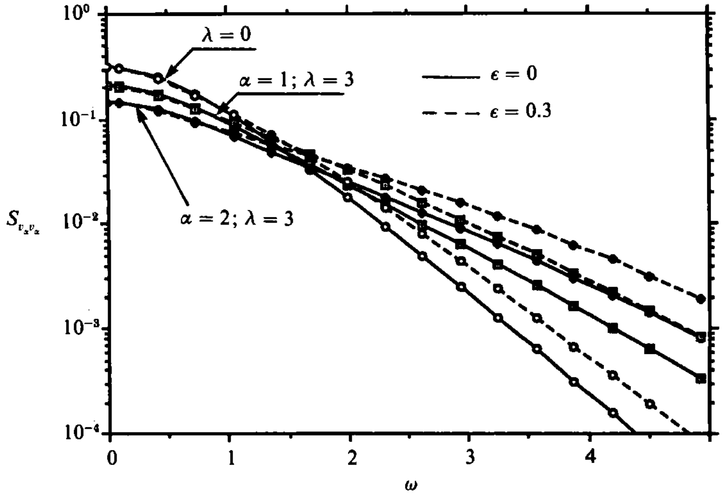


FIGURE 5. Particle velocity spectrum $S_{v_\alpha v_\alpha}(\omega)$ at $\beta = 1$, $\lambda = 0$ and 3.

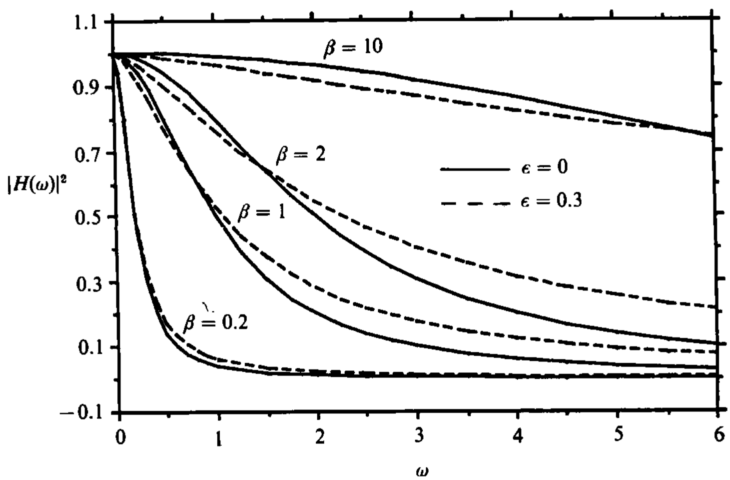


FIGURE 6. Effect of Basset force on the energy transfer function of particle at various β.

are virtually independent of ϵ, the differences in $R_{v_\alpha v_\alpha}(\tau)$ and $S_{v_\alpha v_\alpha}(\omega)$ for $\epsilon = 0$ and $\epsilon = 0.3$ must be due to differences in $H(\omega)$, the frequency response function defined in (10*b*). Figure 6 shows the particle energy transfer function, $|H(\omega)|^2$, as a function of ω for β ranging from 0.2 to 10. It can be seen that the response at high frequency is significantly changed by the Basset force. Thus, one can conclude that the Basset force does influence the particle motion, and that this influence comes from a change in $H(\omega)$ or $|H(\omega)|^2$. It is apparent from (12) and from figure 6 that only the high-frequency component of the energy spectrum of the particle velocity is significantly affected by the Basset force. This is also indicated in figure 4 and figure 5. It can be seen that increasing λ increases the change in $S_{v_\alpha v_\alpha}(\omega)$ because, as mean drift velocity increases, the particle sees a more rapidly evolving flow field. It, thus, experiences more energetic turbulence at higher frequency than it would at small or zero λ.

The quantity influenced most by the Basset force is the intensity of particle motion, $\langle v_\alpha^2 \rangle$, that is important in problems involving particle deposition and involving erosion by particle impact on walls. Figure 7 shows the relative change in

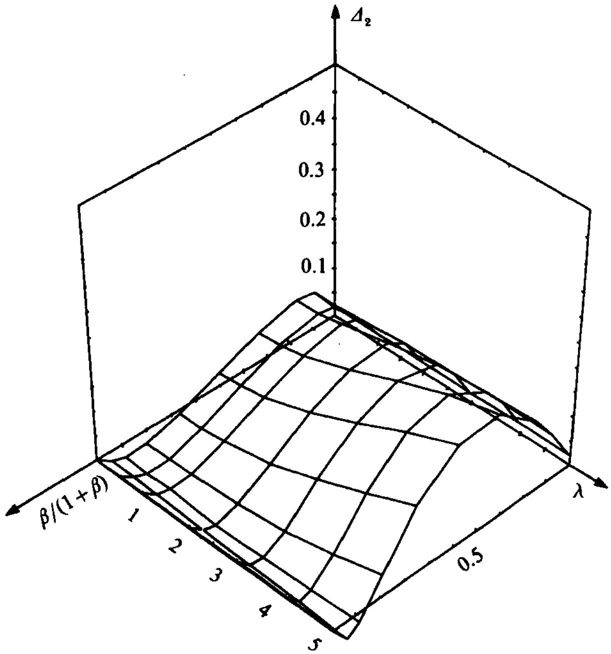
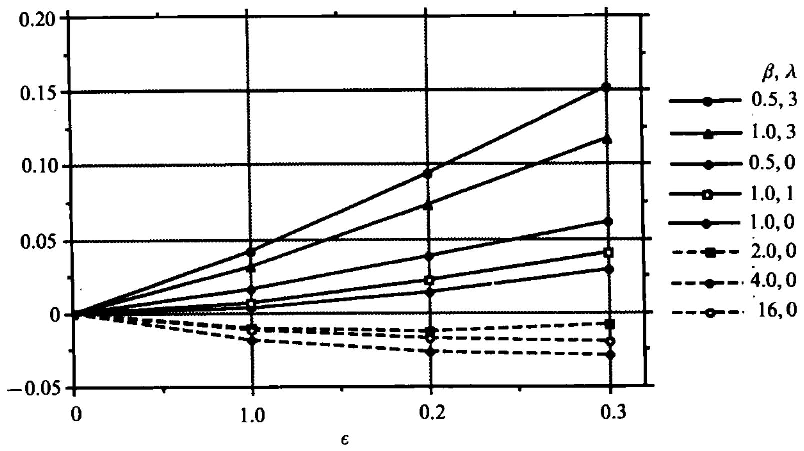


FIGURE 7. Relative difference of particle lateral intensity $\Delta_2 = (\langle v_2^2(\epsilon)\rangle - \langle v_2^2(\epsilon = 0)\rangle)/\langle v_2^2(\epsilon = 0)\rangle$ as a function of β and λ at $\epsilon = 0.3$.

the lateral intensity of the particle fluctuation, $\Delta_2 = (\langle v_\alpha^2(\epsilon = 0.3)\rangle - \langle v_\alpha^2(\epsilon = 0)\rangle)/\langle v_\alpha^2(\epsilon = 0)\rangle$, as a function of β and λ at $\epsilon = 0.3$. The effect of λ is obvious. As λ increases, Δ_2 increases because of the increase in $S^{\mathrm{p}}_{u_\alpha u_\alpha}(\omega)$ at high frequency. The effect of crossing-trajectory therefore is to amplify the Basset force. The effect of inertia, β, on Δ_2 is more complex. The relative difference, Δ_2, approaches zero as $\beta \to 0$ or as $\beta \to \infty$. For small values of β, $\Delta_2 > 0$, since the Basset force tends to increase the turbulence intensity of large particles. For larger values of β, e.g. $\beta > 2$ for $\lambda = 0$, one has $\Delta_2 < 0$. This indicates that the Basset force actually reduces the intensities of motion for a small particle. This can be seen more clearly from figure 8(*a*) where Δ_2 is shown as a function of ϵ for various values of β and λ. The result that $\Delta_2 < 0$ for larger values of β reflects the competition between two factors appearing in the Basset term in (6), the excitation related to $\boldsymbol{u}^{\mathrm{p}}(t)$ and response related to $\boldsymbol{v}(t)$. The factor controlling the competition is mainly the inertia of the particle. For $\epsilon = 0$, if $\beta \gg 1$, the particle energy transfer function is nearly flat. Thus, the particle is able to respond to nearly all frequencies of the turbulent fluctuations sensed along its trajectory. When the Basset term is included the particle becomes much more sluggish than at $\epsilon = 0$ for $\beta \gg 1$, so it picks up less energy. Thus, even with additional excitation from the Basset term, the particle effectively receives less energy from turbulence. On the other hand, for small β, the particle is already very sluggish in responding to turbulence excitation. Additional inertia due to the Basset term does not change the sluggishness of the particle very much. However, the additional increase in excitation associated with the Basset term is effectively seen and picked up by the particle.

From figure 8(*a*) and figure 8(*b*), one can see that the relative change in longitudinal intensity, Δ_1, is less than the change in laterial intensity, Δ_2, when $\lambda > 0$. This means that the lateral component of the particle fluctuating velocity is more

(*a*)



(*b*)

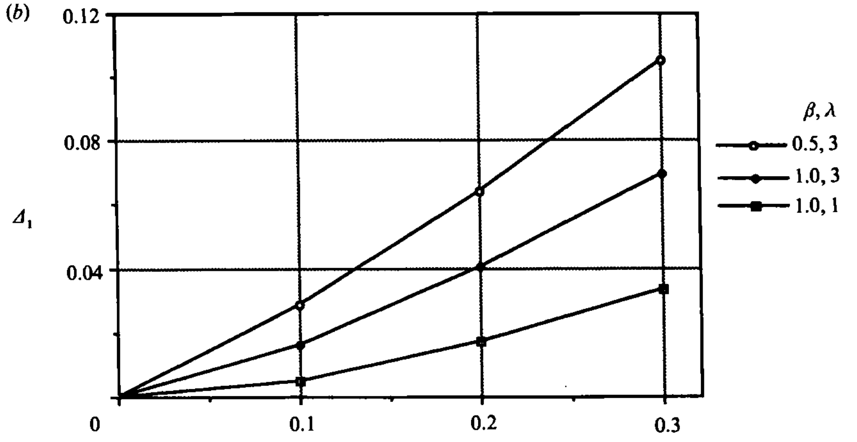


FIGURE 8. Relative difference of particle intensities $\Delta_\alpha = (\langle v_\alpha^2(\epsilon)\rangle - \langle v_\alpha^2(\epsilon = 0)\rangle)/\langle v_\alpha^2(\epsilon = 0)\rangle$ as functions of ϵ at various values of β and λ, (*a*) Δ_1; (*b*) Δ_2.

susceptible to the Basset force than the longitudinal one. Since $|H(\omega)|^2$ is the same for both components, the difference appears in $S^{\mathrm{p}}_{u_\alpha u_\alpha}(\omega)$. As shown in figure 3, $S^{\mathrm{p}}_{u_2 u_2}(\omega)$ is larger than $S^{\mathrm{p}}_{u_1 u_1}(\omega)$ at high frequency for $\lambda > 0$. The particle, therefore, experiences more greatly the change in lateral intensity of the fluid fluctuation due to the effect of crossing-trajectory when the Basset force is included.

As a consequence of the change in the turbulent intensities of particle motion due to the Basset force, the integral timescale of the particle motion, T^{p}_α, is also affected. Since the diffusivities are virtually unchanged when the Basset force is included, any increase in the intensities corresponds to a decrease in the integral timescale that can be computed from $T^{\mathrm{p}}_\alpha = D_{\alpha\alpha}/\langle v_\alpha^2\rangle$.

It is noted that Hinze (1975, p. 468) gave a criterion for neglecting the influence of the Basset force on the motion of particles. For heavy particles (such as solid particles in a gas), the neglect of the Basset-force term affects the amplitude of the oscillating motion of a particle when $(1/2a)(\nu/\omega)^{\frac{1}{2}} \leqslant 0.6$, which is equivalent to $\epsilon' = (\omega a^2/2\nu)^{\frac{1}{2}} \geqslant 0.589$ for the Stokes number, ϵ', based on the oscillating frequency. However, the calculation that lead to the above criterion was based purely on the behaviour of $|H(\omega)|^2$, the particle response function. The effect of changes in the power spectrum, $S^{p}_{u_\alpha u_\alpha}(\omega)$, on the particle motion was not included because the behaviour of $S^{p}_{u_\alpha u_\alpha}(\omega)$ was not known, even approximately, in the presence of the Basset force. The effect of settling rate was not dealt with either. As pointed out by Hinze, this criterion is strictly applicable only to very fine particles for which the particle intensity and diffusivity are close to those of the fluid. The present analysis gives an account of the effect of the Basset force on the turbulence intensities and diffusivities of particle motion with arbitrary inertia and settling rate.

It should be mentioned here that, in practice, $\epsilon = 0.3$ is large since the use of (1) is restricted to low Reynolds number. For example, consider a spherical water droplet of 280 μm in diameter in the central region of a vertical 2 in. pipe with an air flow at $Re \approx 30000$. For this flow, $u_0 \approx 35.6$ cm/s, the fluid diffusivity is $D_f \approx 9.15\ \mathrm{cm^2/s}$, and the typical frequency ω_0 is about $0.913u_0^2/D_f \approx 126/\mathrm{s}$. This gives a Stokes number of $\epsilon = 0.286$. However, the particle Reynolds number based on terminal velocity is about 20, that is far too large for the Stokes law of drag to be accurate. For $\epsilon \approx 0.3$, it is probably more important to consider the effect of a nonlinear drag law because unsteady forces are small.

Before concluding the discussion of the effect of the Basset force, it is necessary to address an issue that has been raised by Reeks & Mckee (1984). They reported that the initial velocity difference that exists between the particle and the fluid at the time when the particle is introduced contributes to the long-time particle diffusivity because the initial disturbance in velocity dies off slowly as $t^{-\frac{1}{2}}$ due to the Basset force while the displacement grows as $t^{\frac{1}{2}}$. The product of velocity and displacement results in a finite contribution to the long-time diffusivity. The unphysical quality of this result potentially challenges the validity of the Basset term. Sano (1981) studied the impulsively started motion of a sphere at low Reynolds number for both small and large times using the complete Navier–Stokes equation. The initial disturbance was found to decay as t^{-2} at large time rather than $t^{-\frac{1}{2}}$. Recently, Mei, Lawrence & Adrian (1990) considered the unsteady flow over a sphere at finite Reynolds number with small oscillation in the free-stream velocity. A solution of the full Navier–Stokes equation was obtained using a finite-difference method. Agreement with the classical result was obtained at high frequencies in that the Basset force was found to vary as $\omega^{\frac{1}{2}}$. However, it was found that the 'Basset force' is proportional to the frequency of the oscillation to the first power. This behaviour was found both for very small Reynolds number, $Re = 2aU/\nu = 0.1$, and for a finite Reynolds number, $Re = 40$, as long as the convection term is kept in the computation. This means that the Basset-force term in the time domain has a kernel that decays much faster than $t^{-\frac{1}{2}}$ at large time because of the influence of convection terms on a slowly varying flow. In the frequency domain, this long-time behaviour corresponds to a small region near $\omega = 0$ whose size depends on particle Reynolds number. The original expression for the Basset force is used in the present analysis, knowing that initial conditions do not contribute to the long-time diffusivities of the particle motion. The present analysis avoids this unphysical behaviour by enforcing the stationarity of the random processes $v(t)$ and $y(t)$ in deriving (16). This suppresses any possible influence of the initial conditions. Consequently, diffusivities obtained here are solely due to the random motions of the particle.

6. Summary and conclusion

(i) An analysis is presented for particle dispersion in isotropic turbulence that includes the effect of the Basset history force, the effect of inertia, and the effect of the crossing of trajectories caused by mean settling velocity. Results are obtained for particle motions in the Stokes regime. Corrsin's conjecture is assumed.

(ii) The analyses of Reeks (1977) and of Pismen & Nir (1978) are shown to be easily extended to the case of density ratio of $O(\epsilon^{-1})$ or higher when the Stokes number is small. The analysis is then accurate up to $O(1)$ when the Basset force is neglected.

(iii) Analysis of dispersion in the absence of the Basset force reveals that diffusivities of particle motion depend strongly on the settling velocity and only weakly on the inertia. The intensities, however, depend strongly on inertia and only weakly upon the settling velocity.

(iv) The Basset force has virtually no influence either on the long-time particle diffusivities or on the detailed structure of the time history of the fluid velocity seen by the particle on its trajectory, in a statistical sense.

(v) The crossing of trajectories tends to enhance the effect of the Basset force by increasing the intensities of turbulent velocities of particle. Particle inertia enhances the effect of the Basset force on the turbulence intensities of particles with large, but finite inertia. It reduces the turbulence of small particles with small inertia.

This work is supported by NSF through CBT 88-00980 and the Multiphase Flow Research Institute through Argonne Contract No. 82862403. We thank the reviewers for their comments on this work.

REFERENCES

AUTON, T. R., HUNT, J. C. R. & PRUD'HOMME, M. 1988 *J. Fluid Mech.* **197**, 241–257.

BASSET, A. B. 1888 *A Treatise on Hydrodynamics*, vol. 2. Dover.

CHAO, B. T. 1964 *Osterreichisches Ingenieur-Archiv.* **18**, 7–21.

CORRSIN, S. 1959 *Adv. Geophys.* **6**, 161–184.

GOUESBET, G., BERLEMONT, A. & PICART, A. 1984 *Phys. Fluids* **27**(4), 827–837.

HINZE, J. O. 1975 *Turbulence.* McGraw-Hill.

KRAICHNAN, R. H. 1970 *Phys. Fluids* **13**, 22–31.

LANDAU, L. E. & LIFSHITZ, E. M. 1959 *Fluid Mechanics.* Pergamon.

LUMLEY, J. L. 1957 Some problems connected with the motion of small particles in turbulent fluid. PhD thesis, The Johns Hopkins University, Baltimore, MD.

LUNDGREN, T. S. & POINTIN, Y. B. 1976 *Phys. Fluids* **19**, 355–358.

MAXEY, M. R. 1987 *J. Fluid Mech.* **174**, 441–465.

MAXEY, M. R. & RILEY, J. 1983 *Phys. Fluids* **26**(4), 863–889.

MEI, R., LAWRENCE, C. J. & ADRIAN, R. J. 1990 Unsteady drag on a sphere at finite Reynolds number with small fluctuations in the free-stream velocity. Submitted.

NIR, A. & PISMEN, L. M. 1979 *J. Fluid Mech.* **94**, 369–381.

PHYTHIAN, R. 1975 *J. Fluid Mech.* **67**, 145–153.

PISMEN, L. M. & NIR, A. 1978 *J. Fluid Mech.* **84**, 193–206.

REEKS, M. W. 1977 *J. Fluid Mech.* **83**, 529–546.

REEKS, M. W. & MCKEE, S. 1984 *Phys. Fluids* **27**(7), 1573–1582.

SANO, T. 1981 *J. Fluid Mech.* **112**, 433–441.

SHLIEN, D. J. & CORRSIN, S. 1974 *J. Fluid Mech.* **62**, 255–271.

TCHEN, C. M. 1947 Mean value and correlation problems connected with the motion of small particles suspended in a turbulent fluid. PhD thesis, Delft University, Netherlands.